Math 103 Day 14: Limits at Infinity

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Outline

Definition

Let f be a function defined on some interval (a, ∞). Then

$$lim_{x \to \infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

Definition

Let f be a function defined on some interval $(-\infty, a)$. Then

$$lim_{x \to -\infty} f(x) = L$$

means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large negative.

Definition

The line $y = L$ is called the **horizontal asymptote** of the curve $y = f(x)$ if either

$$lim_{x \to \infty} f(x) = L \ \ or \ \ lim_{x \to -\infty} f(x) = L$$

Theorem

If $r > 0$ is a rational number, then

$$lim_{x \to \infty} \frac{1}{x^r} = 0$$

If $r > 0$ is a rational number such that x^r is defined for all x, then

$$lim_{x \to -\infty} \frac{1}{x^r} = 0$$